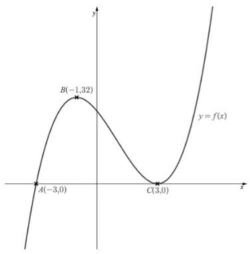
Q1

1

The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points and intercepts with the x-axis are marked on the diagram.



On separate diagrams, sketch the graphs with equations

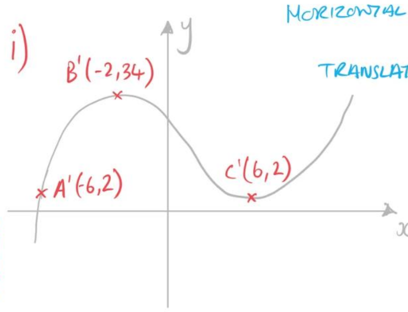
(i) $y = \mathrm{f}\left(\frac{1}{2}x\right) + 2$,
(ii) $y = -\mathrm{f}(x-1)$.

On each diagram, mark the coordinates of the images of the points A, B and C under the given transformation.

[6]

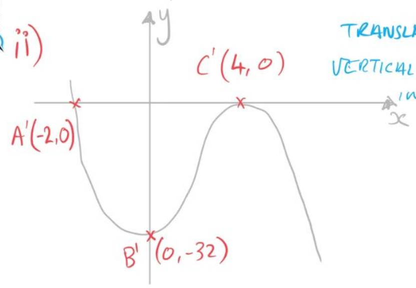
i)

HORIZONTAL STRETCH SF 2
TRANSLATION $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$



ii)

TRANSLATION $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
VERTICAL REFLECTION IN x AXIS



save my exams

Q2

2

Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = -\mathrm{f}(3x-1)$,
(ii) $y = 2\mathrm{f}(5-x)$.

[4]

DEAL WITH INSIDE BRACKETS FIRST

INSIDE = HORIZONTAL (OPPOSITE, TRANSLATION FIRST)
OUTSIDE = VERTICAL

i) TRANSLATION $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

HORIZONTAL STRETCH SF $\frac{1}{3}$

VERTICAL REFLECTION IN x AXIS $(y=0)$

ii) TRANSLATION $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

HORIZONTAL REFLECTION IN y AXIS $(x=0)$

VERTICAL STRETCH SF 2

save my exams

Q3

3

Given that $f(x) = \ln(2x+1)$ find an expression for $g(x)$, where $g(x)$ is obtained by applying the following sequence of transformations to $f(x)$.

1. Translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$,
2. Horizontal stretch by scale factor $\frac{1}{2}$,
3. Reflection in the x-axis.

[4]

save my exams

1. $f(x) \Rightarrow f(x+3)$

$\ln(2(x+3)+1)$

$\ln(2x+6+1) = \ln(2x+7)$

2. $f(x+3) \Rightarrow f(2x+3)$

$\ln(2(2x)+7) = \ln(4x+7)$

3. $f(2x+3) \Rightarrow -f(2x+3)$

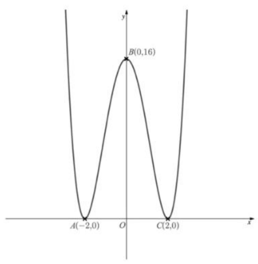
$-\ln(4x+7)$

$$\boxed{g(x) = -\ln(4x+7)}$$

Q4a

4a

A sketch of the graph with equation $y = f(x)$, where $f(x) = (x^2-4)^2$ is shown below.



The points A, B and C are the points where the graph intercepts the coordinate axes.

(a) Sketch the graph of $y = -3f(2x)$, labelling the images of the three points A, B and C.

[3]

(b) Suggest a combination of at least two transformations that will transform the points A, B and C such that none of them lie on the coordinate axes.
Give your answer in the form of an expression in terms of $f(x)$.

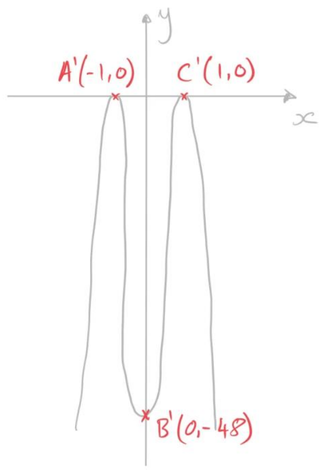
[2]

save my exams

a) HORIZONTAL STRETCH SF $\frac{1}{2}$

VERTICAL STRETCH SF 3

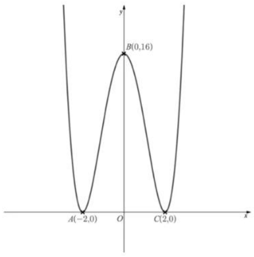
VERTICAL REFLECTION IN x AXIS



Q4b

4b

A sketch of the graph with equation $y = f(x)$, where $f(x) = (x^2 - 4)^2$ is shown below.



The points A, B and C are the points where the graph intercepts the coordinate axes.

(a) Sketch the graph of $y = -3f(2x)$, labelling the images of the three points A, B and C.

[3]

(b) Suggest a combination of at least two transformations that will transform the points A, B and C such that none of them lie on the coordinate axes.
Give your answer in the form of an expression in terms of $f(x)$.

[2]

save my exams

b) TRANSLATION BOTH HORIZONTALLY AND VERTICALLY

ANY VERSION OF $f(x+a)+b$

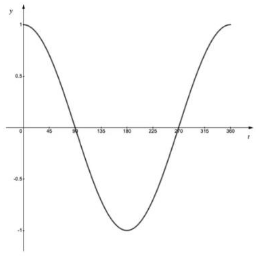
$a \neq \pm 2 \quad b \neq -16 \quad a, b \neq 0$

$\boxed{y = f(x+3) - 4}$

Q5a

5a

The diagram shows the graph of $y = f(t)$, where $f(t) = \cos t,\ 0° \leq x \leq 360°$.



(a) (i) Write down the maximum value of y when $y = -2f(3t)$.
(ii) Write down the value of t for which this maximum occurs.

[2]

(b) Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 2 - 4\sin t,\ 0° \leq x \leq 180°$.

[2]

save my exams

a)

i) VERTICAL STRETCH SF 2 AND REFLECTION

MAX $-2(\cos 3t) = 2$

$\boxed{y = 2}$

ii) MIN REFLECTED UP TO BECOME MAX AT $t = 180$

HORIZONTAL STRETCH SF $\frac{1}{3}$

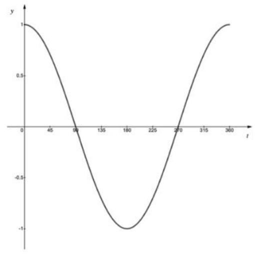
$\frac{180}{3} = 60$

$\boxed{t = 60°}$

Q5b

5b

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \cos t,\ 0° \le x \le 360°$.



(a) (i) Write down the maximum value of y when $y = -2\mathrm{f}(3t)$.
(ii) Write down the value of t for which this maximum occurs.

[2]

(b) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 - 4\sin t,\ 0° \le x \le 180°$.

[2]

save my exams

b)

$f(t) = \cos t$ MULTIPLE OPTIONS

$f(t+90) = -\sin t$ ± 90 HORIZONTAL TRANSLATION

$4f(t+90) = -4\sin t$ VERTICAL STRETCH BY SF 4

$4f(t+90) + 2 = 2 - 4\sin t$ TRANSLATION $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\boxed{4f(t+90)+2}$

Q6

6

The function $\mathrm{f}(x)$ is to be transformed by a sequence of functions, in the order detailed below.

1. A translation by $\binom{2}{0}$
2. A reflection in the y-axis
3. A vertical stretch by scale factor $\frac{2}{3}$
4. A translation by $\binom{0}{4}$

Write down the combined transformation in terms of $\mathrm{f}(x)$.

[3]

save my exams

$f(x)$

1. HORIZONTAL TRANSLATION

$f(x-2)$

2. HORIZONTAL REFLECTION

$f(-x-2)$

3. VERTICAL STRETCH

$\frac{2}{3}f(-x-2)$

4. VERTICAL TRANSLATION

$\boxed{\frac{2}{3}f(-x-2)+4}$

www.mikedemy.com

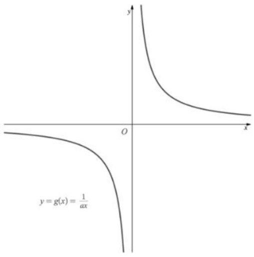
Q7a

7a

The diagram below shows the graph of $y = g(x)$ where

$$g(x) = \frac{1}{ax}, \qquad a, x \neq 0$$

where a is a constant.



(a) (i) Write down the equations of the asymptotes on the graph of $y = g(x)$.
(ii) Determine the equations of the asymptotes on the graph of $y = 3g(2x + 1)$.

[5]

(b) Determine the domain and range of the series of transformations to $y = f(x)$ where $f(x) = -2g\left(\frac{1}{3}x + 3\right) - 4$.

[3]

a)

i) $\boxed{x = 0 \quad y = 0}$

ii) x AFFECTED BY TRANSLATION $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
FIRST THEN STRETCH SF $\frac{1}{2}$
OR SOLVE $2x + 1 = 0$

$x = 0 \Rightarrow x = -1 \Rightarrow x = -0.5$

y AFFECTED BY STRETCH SF 3

$y = 0 \Rightarrow y = 0$

$\boxed{x = -0.5 \quad y = 0}$

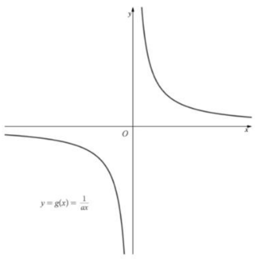
Q7b

7b

The diagram below shows the graph of $y = g(x)$ where

$$g(x) = \frac{1}{ax}, \qquad a, x \neq 0$$

where a is a constant.



(a) (i) Write down the equations of the asymptotes on the graph of $y = g(x)$.
(ii) Determine the equations of the asymptotes on the graph of $y = 3g(2x + 1)$.

[5]

(b) Determine the domain and range of the series of transformations to $y = f(x)$ where $f(x) = -2g\left(\frac{1}{3}x + 3\right) - 4$.

[3]

b) DOMAIN $g(x) \neq 0$ (x ASYMPTOTE)
TRANSLATION $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ STRETCH SF 3
OR SOLVE $\frac{1}{3}x + 3 = 0$

$\boxed{\text{DOMAIN } x \in \mathbb{R} \quad x \neq -9}$

RANGE $g(x) \neq 0$ (y ASYMPTOTE)
STRETCH AND REFLECT SF -2
TRANSLATE $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$\boxed{\text{RANGE } f(x) \in \mathbb{R} \quad f(x) \neq -4}$